Reading Time: An initial **2 minutes** to view **BOTH** sections

MATHEMATICS METHODS : UNITS 3 & 4, 2023

Test 1 – Differentiation Rules and Applications (10%)

3.1.7, 3.1.8, 3.1.10 – 3.1.16, 3.2.1 – 3.2.3

RG

Time Allowed	First Name	Surname	Marks
30 minutes	MARKING GUIDE		27 marks

Circle your Teacher's Name: Mrs Alvaro, Ms Chua, Mrs Fraser-Jones, Mrs Greenaway, Mr Luzuk, Mrs Murray, Ms Narendranathan, Mr Tanday

Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Not Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

PART A – CALCULATOR FREE

QUESTION 1 **(4 marks)**

Find the derivative of $y=(3x^2-2x)^3$, **clearly demonstrating the use of the chain rule**. DO NOT SIMPLIFY.

$y=(3x^2-2x)^3$

$u=3x^2-2x$ ✓ Shows substitution

$\frac{du}{dx}=6x-2$

$y=u^3$

$\frac{dy}{du}=3u^2$ ✓ Finds derivatives

$\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$ ✓ Uses chain rule

$=3u^2(6x-2)$

$=3(3x^2-2x)^2(6x-2)$ ✓ Final expression for derivative

QUESTION 2 **(2, 2 – 4 marks)**

Find the derivative of the following with respect to x (DO NOT SIMPLIFY):

a) $y=x^3(3x-5)^4$

$y'=3x^2(3x-5)^4+4(3x-5)^3(3)(x^3)$

✓ Use of product rule evident

✓ Correct expression

b) $f(x)=\pi^2+\sqrt{x^2-3x}$

$y=(x^2-3x)^{\frac{1}{2}}$

$y'=\frac{1}{2}(x^2-3x)^{-\frac{1}{2}}(2x-3)$

✓ Use of power/chain rule evident

✓ Correct expression

QUESTION 3 **(3 marks)**

Find the gradient of the curve with the equation $y=\frac{2x^2-1}{x^2+2}$ where $x=2$.

$$y'=\frac{4x(x^2+2)-(2x)(2x^2-1)}{(x^2+2)^2}$$ ✓ Use of quotient rule evident

$$y'(2)=\frac{4(2)(4+2)-(4)(8-1)}{(4+2)^2}$$ ✓ Substitution

$$=\frac{48-28}{36}$$

$$=\frac{5}{9}$$ ✓ Correct value

OR

$$y=(2x^2-1)(x^2+2)^{-1}$$

$$y'=(4x)(x^2+2)^{-1}+(-1)(x^2+2)^{-2}(2x)(2x^2-1)$$ ✓ Use of product rule evident

$$y'(2)=(8)(6)^{-1}-(6)^{-2}(4)(7)$$ ✓ Substitution

$$=\frac{8}{6}-\frac{28}{36}$$

$$=\frac{5}{9}$$ ✓ Correct value

QUESTION 4 **(1, 3 – 4 marks)**

For $y=\frac{3x^2+8}{2x}$:

a) Find $\frac{dy}{dx}$ (DO NOT SIMPLIFY)

$$\frac{dy}{dx}=\frac{(6x)(2x)-2(3x^2+8)}{(2x)^2}$$ ✓ Correct equation

b) State the approximate increase in y (in terms of p) as x increases from 2 to $2+p$ when p is small.

$$\delta y=\frac{dy}{dx}\delta x$$

$x=2$

$\delta x=p$

✓ Use of increments formula

$$=\frac{6\times2\times2\times2-2(12+8)}{4\times4}p$$

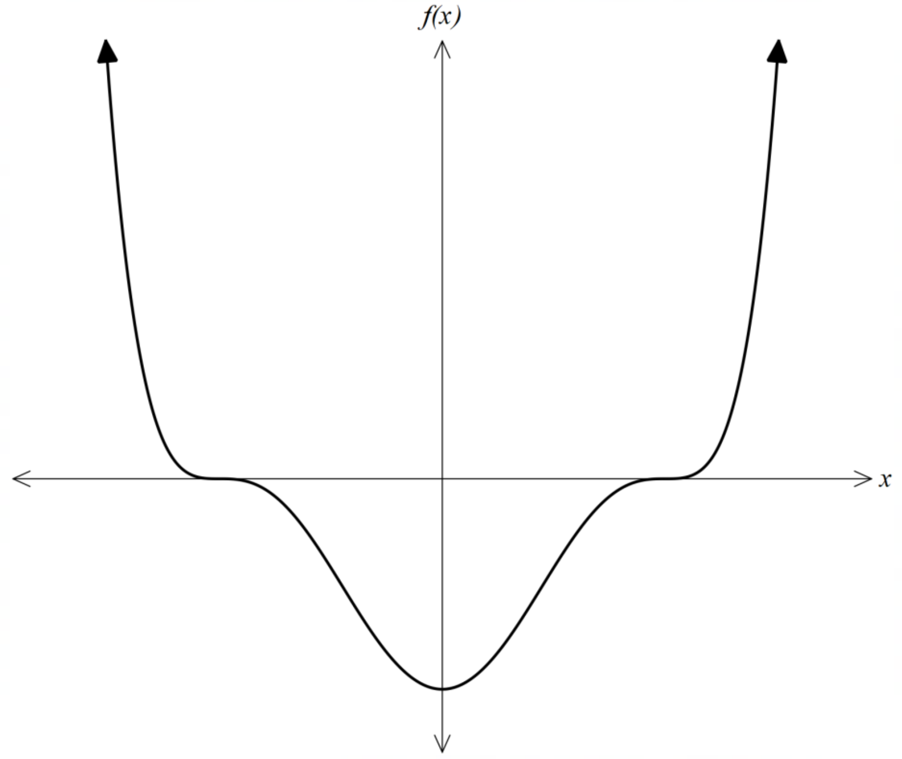
$$=\frac{8}{16}p$$

$$=\frac{p}{2}$$ ✓ Substitutes x and δx

Therefore y increases by approximately $\frac{p}{2}$. ✓ States decrease

QUESTION 5 **(3 marks)**

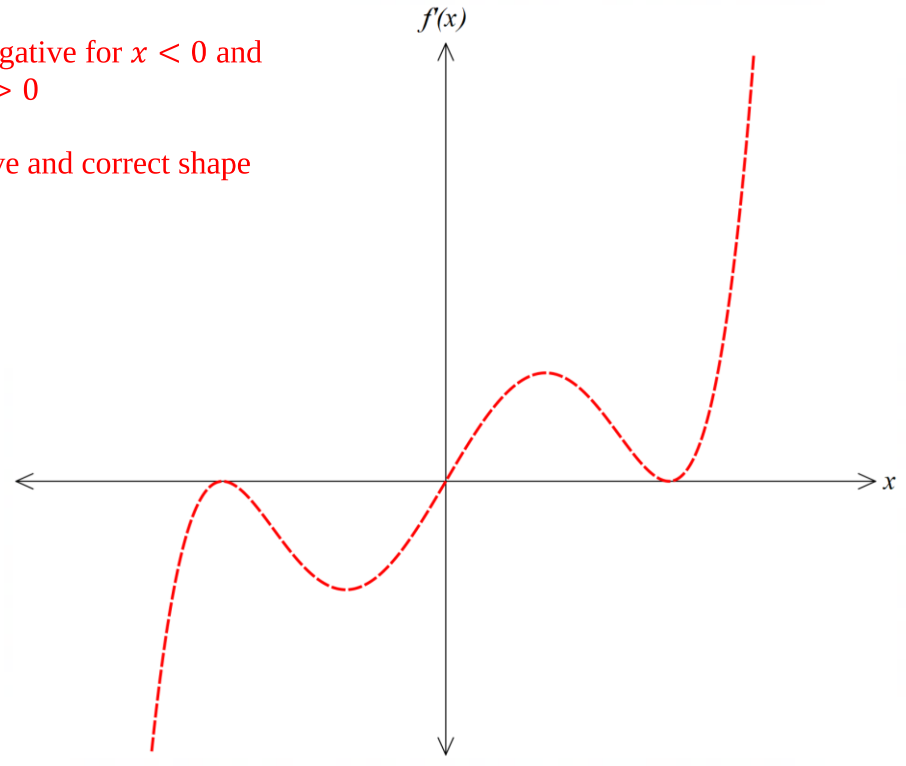
The graph of $y = f(x)$ is as shown below. On the axes provided, sketch the graph of $y = f'(x)$



✓ Stationery points correctly lined up

✓ Gradient negative for $x < 0$ and positive for $x > 0$

✓ Smooth curve and correct shape



QUESTION 6 **(4 marks)**

A variable z is defined as the sum of the squares of two other variables x and y. That is, $z = x^2 + y^2$. Furthermore $x + y = 4$. Find the values of x and y such that z takes its minimum value.

$z = x^2 + (4 - x)^2$ ✓ Writes z in terms of x

$\frac{dz}{dx} = 2x + 2(4 - x)(-1)$

$= 2x - 2(4 - x)$ ✓ Finds $\frac{dz}{dx}$

$0 = 2x - 8 + 2x$

$= 4x - 8$

$x = 2$ ✓ Equates to 0 and solves for x

$y = 2$ ✓ States y value

QUESTION 7 **(1, 2, 2 – 5 marks)**

Find the following:

a) $\int x^2 - 3x + 2dx = \frac{x^3}{3} - \frac{3x^2}{2} + 2x + c$

✓ Correct expression

b) $\int x - x^{\frac{1}{2}} dx = \frac{x^2}{2} - \frac{2x^{\frac{3}{2}}}{3} + c$

✓ One term correct
✓ Both terms correct

NB: Penalise if 2nd term left over 3/2

c) $\int x^{2a+2} dx = \frac{x^{2a+3}}{2a+3} + c$

✓ Numerator correct
✓ Denominator correct